

“Civilization as we know it is based on ten ideas. These are Newton’s three laws, the three laws of thermodynamics and Maxwell’s equations. Everything that you see around you which differentiates modern times from the past is based on these concepts. Soon, Schrodinger’s equation will join these ten ideas. People will tell you that civilization is about art, or literature, or architecture... but these components of civilization have more or less been the same for thousands of years. The only real difference between today and thousands of years ago is physics. If civilization collapsed tomorrow, we could rebuild it in the same fashion armed with these ideas.”

- Prof. Seamus C. Davis, U.C. Berkeley.

If you have any questions, suggestions or corrections to the solutions, don’t hesitate to e-mail me at dfk@uclink4.berkeley.edu!

Problem 1 Rohlf 8.41

Here we wish to show that the average value of $1/r$ is independent of the orbital angular momentum for the hydrogen atom. This result is straightforward if we introduce the virial theorem (see, e.g., B.H. Bransden and C.J. Joachain, **Introduction to Quantum Mechanics**, pgs. 227-228), which states (in one particular form) that for a spherically symmetric potential $V(r) \propto r^n$ one has for a stationary state:

$$2\langle T \rangle = n\langle V(r) \rangle,$$

where T is the kinetic energy. This is analogous to a classical result of the same name. The average potential energy for hydrogen is proportional to $\langle 1/r \rangle$, as is the kinetic energy and hence the total energy from the virial theorem. We know that the total energy for the hydrogen atom (from the Bohr model) is independent of l and depends only on the principal quantum number n . Consequently, $\langle 1/r \rangle$ is also independent of the orbital angular momentum.

Problem 2 Rohlf 9.3

We seek a totally antisymmetric wavefunction Ψ for 3 electrons in terms of $\psi_a(\mathbf{r}_1)$, $\psi_b(\mathbf{r}_2)$ and $\psi_c(\mathbf{r}_3)$. The wavefunction must be totally antisymmetric because we have three identical fermions. Such a wavefunction is given below. It is easily verified that under particle interchange it flips sign.

$$\Psi = \frac{1}{\sqrt{3!}} [\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)\psi_c(\mathbf{r}_3) - \psi_a(\mathbf{r}_1)\psi_c(\mathbf{r}_2)\psi_b(\mathbf{r}_3) + \psi_b(\mathbf{r}_1)\psi_c(\mathbf{r}_2)\psi_a(\mathbf{r}_3) - \psi_b(\mathbf{r}_1)\psi_a(\mathbf{r}_2)\psi_c(\mathbf{r}_3) + \psi_c(\mathbf{r}_1)\psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_3) - \psi_c(\mathbf{r}_1)\psi_b(\mathbf{r}_2)\psi_a(\mathbf{r}_3)] \quad (1)$$

Problem 3 Rohlf 9.10

An atom has two electrons in the d -subshell. What are the possible values of total z -angular momentum?

Well, we know that the total orbital angular momentum l_{tot} can range from $l_1 + l_2$ to $|l_1 - l_2|$. Thus $l_{\text{tot}} = 4, 3, 2, 1, 0$. If l_{tot} is even, then the spatial part of the wavefunction Ψ_{spatial} is symmetric, and if l_{tot} is odd, then Ψ_{spatial} is antisymmetric.

With two electrons, the total spin can be $s_{\text{tot}} = s_1 + s_2 = 1, 0$. If $s_{\text{tot}} = 1$ then the spin function Ξ_{spin} is symmetric. The total wavefunction $\Phi_{\text{total}} = \Psi_{\text{spatial}} \cdot \Xi_{\text{spin}}$ must be antisymmetric since we are dealing with identical fermions. Therefore if $s_{\text{tot}} = 1$ then Ψ_{spatial} must be antisymmetric, meaning that l_{tot} is odd (1 or 3). In this case the total angular momentum j can take on the values $j = 4, 3, 2, 1, 0$.

If $s_{\text{tot}} = 0$ then the spin function Ξ_{spin} is antisymmetric, and Ψ_{spatial} must be symmetric. In this case l_{tot} is even (0, 2 or 4). The possible values of j are 0, 2 or 4.

The largest j value possible is 4, so the possible m_j values are:

$$m_j = -4, -3, -2, -1, 0, 1, 2, 3, 4$$

Problem 4 Rohlf 9.25

(a)

Sodium atoms are placed in a magnetic field of 1.5 T. The Zeeman splitting of the ground state ($n = 0, l = 0$) is given by the shift of energy due to the different spin states the single valence electron can have. Energy shifts are

$$E_{\pm} = \pm \mu_B B$$

where μ_B is the Bohr magneton. The numerical value for the splitting is given by

$$\Delta E = 2\mu_B B = 2(6 \times 10^{-5} \text{ eV/T})(1.5 \text{ T}) = 1.8 \times 10^{-4} \text{ eV}.$$

(b)

If 1/3 of the sodium atoms are in the higher energy state, then the Boltzmann factor

$$e^{-\Delta E/(kT)} = \frac{1/3}{2/3} = 1/2.$$

From which we calculate

$$kT = \frac{\Delta E}{\ln 2} = 2.5 \times 10^{-4} \text{ eV}.$$

Hence,

$$\boxed{T \approx 2.9 \text{ K}}$$

(c)

Same as above:

$$e^{-\Delta E/(kT)} = 49/51.$$

From which we calculate

$$kT = 4.4 \times 10^{-3} \text{ eV}$$

and

$$\boxed{T \approx 51 \text{ K}}$$

Problem 5 *Rohlf 9.31*

A sample of Na atoms are placed in a 1.0 T magnetic field. We calculate the energy shifts for the $3s_{1/2}$, $3p_{1/2}$ and $3p_{3/2}$ states. Note that in this case $\mu_B B = 6 \times 10^{-5}$ eV/T.

For the $3s_{1/2}$ state ($s = 1/2$, $l = 0$, and $j = 1/2$), the Lande factor is given by:

$$g_L = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} = 2.$$

Therefore the energy shifts, given by

$$\Delta E = \mu_z B_z = g_L \mu_B B m_j$$

are

$$\Delta E = \pm \mu_B B.$$

Similarly, for the $3p_{1/2}$ state, $g_L = 2/3$ so

$$\Delta E = \pm \frac{1}{3} \mu_B B.$$

For the $3p_{3/2}$ states $g_L = 4/3$ so

$$\Delta E = \pm \frac{2}{3} \mu_B B, \quad \pm 2 \mu_B B.$$